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UNITARITY, (ANTI)SHADOWING AND THE BLACK (GRAY) DISC LIMIT

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Abstract By using realistic models for elastic hadron scattering we demonstrate that at present accelerator energies the s-channel unitarity bound is safe, not to be reached until 10^5 GeV, while the black disc limit is saturated around 1 TeV. It will be followed by a larger transparency (grayness) of the scattered particles.

1 Introduction

Our decision to write this paper was motivated partly by recent claims that in high energy hadron scattering the black disc limit has been reached and the violation of the s-channel unitarity in some models is just around the corner. While the first statement is true and has interesting physical consequences, the second one is wrong for any realistic model fitting the existing data on proton and antiproton scattering up to highest accelerator energies.

To start with, let us remind the general definitions and notations.
Unitarity in the impact parameter (b) representation reads

$$\Im\mathrm{m}h(s,b) = |h(s,b)|^2 + G_{in}(s,b), \tag{1}$$

where $h(s,b)$ is the elastic scattering amplitude at $\sqrt{s}$ center of mass energy (with $\Im\mathrm{m}h(s,b)$ usually referred as the profile function, representing the hadron opacity) and $G_{in}(s,b)$, called the inelastic overlap function, is the sum over all inelastic channel contributions. Integrated over $\vec{b}$, (1) reduces to a simple relation between the total, elastic and inelastic cross sections $\sigma_{tot}(s) = \sigma_{el}(s) + \sigma_{in}(s)$.

Equation (1) imposes an absolute limit

$$0 \le |h(s,b)|^2 \le \Im\mathrm{m}h(s,b) \le 1, \tag{2}$$

while the so-called "black disc" limit $\sigma_{el}(s) = \sigma_{in}(s) = \frac{1}{2}\sigma_{tot}(s)$ or

$$\Im\mathrm{m}h(s,b) = 1/2 \tag{3}$$

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is a particular realization of the optical model, namely it corresponds to the maximal absorption within the eikonal unitarization, when the scattering amplitude is approximated as

$$h(s,b) = \frac{i}{2}(1 - \exp[\mathrm{i}\omega(\mathrm{s,b})]\,), \tag{4}$$

with a purely imaginary eikonal $\omega(s,b) = i\Omega(s,b)$.

Eikonal unitarization corresponds to a particular solution of the unitarity equation

$$h(s,b) = \frac{1}{2}\left[1 \pm \sqrt{1 - 4G_{in}(s,b)}\right], \tag{5}$$

the one with minus sign.

The alternative solution, that with plus sign is known [1, 2] and realized within the so-called U-matrix [4] approach [3, 4] where the unitarized amplitude is

$$h(s,b) = \frac{\Im\mathrm{m}U(s,b)}{1 - i\;U(s,b)}, \tag{6}$$

where now U is the input "Born term", the analogue of the eikonal ω in (4).

In the U-matrix approach, the scattering amplitude $h(s,b)$ may exceed the black disc limit, the colliding particles becoming progressivly more transparent (gray) as the energy increases. The transition from a "black" to a "gray" disc corresponds to the transition from shadowing to antishadowing [1]. We shall present a particular realization of this regime in Sec. 4.

The impact parameter amplitude may be calculated either directly from the data, as it was done *e.g.* in [5, 6] (where, however, the real part of the amplitude was neglected) or by using a particular model that fits the data sufficiently well. There are several models appropriate for this purpose. In a classical paper [7] on this subject, from the behaviour of $G_{in}(s,b)$ with the energy, the proton is characterized as getting "BEL" (Blacker, Edgier and Larger). As anticipated in the title of our paper, the proton, after having reached its maximal darkness around the Tevatron energy region, may get less opaque beyond.

Actually, the construction of any scattering amplitude rests on two premises : the choice of the input, or "Born term" and the relevant unitarization procedure (eikonal or U-matrix in our case). Within the present accelerator energy region there are several models that fit the data reasonably well. Compatible within the region of the present experiments, they differ significantly when extrapolated to higher energies. We shall consider two representative examples, namely the Donnachie-Landshoff (D-L) model [8, 9] and the dipole Pomeron (DP) model [4, 10])

In Sec. 2 we present the necessary details about the two realistic models (D-L, DP), then, focusing on the DP model, we investigate in Sec. 3 the unitarity properties at the "Born level" and in Sec. 4 we study the optical properties (transparency) after unitarization; a comparaison with the D-L model is given in appendix.

[4]We follow traditional terminology, although the word "matrix" in this context is misleading, since U, similar to the eikonal, is a single function rather than a matrix.

2 The "Born term"

The Donnachie-Landshoff (D-L) model [8] is popular for its simplicity. Essentially, it means the following four-parametric empirical fit to all the total hadronic cross sections

$$\sigma_{tot} = X\ s^{\delta} + Y\ s^{\delta_r}, \tag{7}$$

where two of the parameters, namely $\delta = \alpha_P(0) - 1 \approx 0.08$, and $\delta_r (< 0)$ are universal. While the violation of the Froissart-Martin (F-M) bound,

$$\sigma_{tot}(s) < C\ (\ell n s)^2 \quad C = 60 \text{ mb}\ , \tag{8}$$

inherent in that model, is rather an aesthetic than a practical defect (because of the remoteness of the energy where eventually it will overshoot the F-M limit), other deficiencies of the D-L model (or any other model based on a supercritical Pomeron) are sometimes criticized in the literature, but so far nobody was able to suggest anything significantly better instead. A particular attractive feature of the D-L Pomeron, made of a single term, is its factorizability, although this may be too crude an approximation to reality.

The t dependence in the Donnachie-Landshoff model is usually chosen [9] in the form close to the dipole formfactor. For the present purposes a simple exponential residue in the Pomeron amplitude will do as well, with the signature included

$$A(s,t) = -\ N \left(-i\frac{s}{s_{dl}}\right)^{\alpha(t)} e^{Bt}\ , \tag{9}$$

where $\alpha(t) = \alpha(0) + \alpha'\ t$ is the Pomeron trajectory and N is a dimensionless normalization factor related to the total cross section at $s = s_{dl}$ by the optical theorem

$$N = \frac{s_{dl}}{4\pi \sin\frac{\pi}{2}\alpha(0)} \sigma_{tot}(s = s_{dl})\ . \tag{10}$$

According to the original fits [8, 9]: $s_{dl} = 1$ GeV^2, $\alpha(0) = 1.08$, $\alpha' = 0.25$ GeV^{-2}, and $X = 21.70$ mb (see (7)) resulting in $N = \frac{X}{4\pi \sin \pi\alpha(0)/2} = 4.44$. By identifying

$$\frac{d\sigma(s,t)}{dt} = \frac{d\sigma(s,t=0)}{dt}\ e^{B_{exp}(s)\ t} \tag{11}$$

and choosing the CDF or E410 result for the slope B_{exp} at the Tevatron energy, we obtain $B = \frac{1}{2}B_{exp}(s) - \alpha' \ell n \frac{s}{s_{dl}} = 4.75$ GeV^{-2}.

In the dipole Pomeron (DP) model [4], factorizable only at asymptotically high energies, on the other hand, logarithmically rising cross sections are produced at a unit Pomeron intercept only and thus DP does not conflict with the F-M bound. While data on total cross section are compatible with a logarithmic rise (DP with unit intercept) the ratio σ_{el}/σ_{tot} is found (see [11] for details) for $\delta = 0$ to be a monotonically decreasing function of the energy for any physical values of the parameters. The experimentally observed rise of this ratio can be achieved only for $\delta > 0$ and thus requires the introduction of a "supercritical" Pomeron, $\alpha(0) > 1$. As a result, the rise of the total cross sections is driven and shared by the dipole and the "supercritical" intercept. The parameter $\delta = \alpha(0) - 1$ in the DP model is nearly

Table 1: Parameters of the Dipole Pomeron found in [10].

a	b_p	$\alpha(0)$	$\alpha'(\mathrm{GeV}^{-2})$	ϵ	$s_0(\mathrm{GeV}^2)$
355.6	10.76	1.0356	0.377	0.0109	100.0

half that of the D-L model, making it safer in practice from the point of view of the unitarity bounds. Generally speaking, the closer the input to the unitarized output, the better the convergence of the unitarization procedure.

Let us remind that apart from the "conservative" F-M bound, any model should satisfy also s-channel unitarity. We demonstrate below that both the D-L and DP model are well below this limit and will remain so within the forseable future. (Let us remind that the D-L and the DP model are close numerically, although they are different conceptually and consequently they extrapolations to superhigh energies will differ as well.)

The (dimensionless) elastic scattering amplitude corresponding to the exchange of a dipole Pomeron reads

$$\begin{aligned} A(s,t) = & \frac{d}{d\alpha}\left[\mathrm{e}^{-i\pi\alpha/2}G(\alpha)(s/s_0)^{\alpha}\right] \\ = & \ \mathrm{e}^{-i\pi\alpha/2}(s/s_0)^{\alpha}[G'(\alpha)+(L-i\pi/2)G(\alpha)]\ , \end{aligned} \tag{12}$$

where $L \equiv \ell n\dfrac{s}{s_0}$, $\alpha \equiv \alpha(t)$ is the Pomeron trajectory; in this paper, for simplicity we use a linear trajectory $\alpha(t) = \alpha(0) + \alpha' t$.

By identifying $G'(\alpha) = -a\mathrm{e}^{b_p(\alpha-1)}$, (12) can be rewritten in the following "geometrical" form

$$A(s,t) = i\frac{as}{b_p s_0}\left[r_1^2(s)\ \mathrm{e}^{r_1^2(s)[\alpha(t)-1]} - \epsilon\ r_2^2(s)\ \mathrm{e}^{r_2^2(s)[\alpha(t)-1]}\right]\ , \tag{13}$$

where

$$r_1^2(s) = b_p + L - i\frac{\pi}{2}\ ,\ r_2^2(s) = L - i\frac{\pi}{2}\ . \tag{14}$$

The model contains the following adjustable parameters: $a, b_p, \alpha(0), \alpha', \epsilon$ and s_0.

In Table 1 we quote the numerical values of the parameters of the dipole Pomeron fitted in [10] to the data on proton-proton and proton-antiproton elastic scattering :

$$\sigma_{tot}(s) = \frac{4\pi}{s}\Im\mathrm{m}A(s,0)\ ,\ \rho(s) = \frac{\Re\mathrm{e}A(s,0)}{\Im\mathrm{m}A(s,0)}\quad ;\qquad 4 \le \sqrt{s}(\mathrm{GeV}) \le 1800 \tag{15}$$

as well as the differential cross-section

$$\frac{d\sigma(s,t)}{dt} = \frac{\pi}{s^2}\,|A(s,t)|^2\ ;\quad 23.5 \le \sqrt{s}(\mathrm{GeV}) \le 630\ ;\quad 0 \le |t|(\mathrm{GeV}^2) \le 6\ . \tag{16}$$

In that fit, apart from the Pomeron, the Odderon and two subleading trajectories ω and f were also included. Here, for simplicity and clarity we consider only the dominant term at high energy due to the Pomeron exchange with the parameters fitted in [10]. The extent to which this Pomeron is a good approximation in the TeV region is discussed in details in [12].

The quality of this fit is illustrated and discussed in [10]. With such a simple model and small number of parameters, better fits are hardly to be expected.

We use the above set of parameters to calculate the impact parameter amplitude, and to scrutinize in Sec. 3 the unitarity properties of this "Born level" amplitude. In Sec. 4 we introduce a unitarization procedure, necessary at higher energies and discuss the relevant physical consequences.

To summarize, the DP model with a unit intercept is selfconsistent in the sense that its functional (logarithmic) form is stable with respect to unitarization. Moreover, the presence of the second term, proportional to ϵ in (13) has the meaning of absorptions and it is essential for the dip mechanism. It can be viewed also as one more unitarity feature of the model.
In the limit of very high energies, when $L \gg b_p$ the two (squared) radii $R_i^2 = \alpha' r_i^2$ become equal and real and the model obeys exact geometrical scaling as well as factorization (see next section). Alternatively, it corresponds to the case of no absorptions ($\epsilon = 0$).
However attractive, the case of a unit intercept ($\delta = 0$) is only an approximation to the more realistic model, requiring $\delta > 0$ to meet the observed rise of the ratio σ_{el}/σ_{tot}. For such a "supercritical" Pomeron unitarization becomes inevitable.

3 Impact parameter representation, unitarity and the black disc limit

The elastic amplitude in the impact parameter representation in our normalization is

$$h(s,b) \;=\; \frac{1}{2s}\int_0^\infty dq\; qJ_0(bq)A(s,-q^2)\;, \qquad q=\sqrt{-t}\;. \tag{17}$$

The impact parameter representation for linear trajectories [5] is calculable explicitly for the DP model (13)

$$h(s,b) \;= i\; g_0\; [e^{r_1^2\delta}\; e^{-b^2/4R_1^2} \;-\; \epsilon\; e^{r_2^2\delta}\; e^{-b^2/4R_2^2}]\;, \tag{18}$$

where

$$R_i^2 = \alpha' r_i^2 \;\; (i=1,2) \qquad ;\; g_0 = \frac{a}{4b_p\alpha' s_0}\;. \tag{19}$$

Asymptotically (*i.e.* when $L \gg b_p$, *i.e.* $\sqrt{s} \gg 2.$ TeV, with the parameters of Table 1),

$$h(s,b) \underset{s\to\infty}{\longrightarrow} i\; g(s)\; (1-\epsilon)\; e^{-\frac{b^2}{4R^2}}\;, \tag{20}$$

where

$$R^2 = \alpha' L \qquad ;\; g(s) = g_0\left(\frac{s}{s_0}\right)^{\delta}\;. \tag{21}$$

To illustrate the s-channel unitarity, we display in Fig. 1 a family of curves showing the imaginary part of the amplitude in the impact parameter-representation at various energies; also shown is the calculated (from (1)) inelastic overlap function.

[5] Other cases were treated *e.g.* in [4].

Table 2: Central opacity of the nucleon $\Im m h(s, 0)$ calculated at ISR, SPS, Tevatron energies compared with experiment.

$\sqrt{s}$	53 GeV	546 GeV	1800 GeV
exp	0.36[6]	0.420 ± 0.004 [13]	0.492 ± 0.008 [14]
th	0.36	0.424	0.461

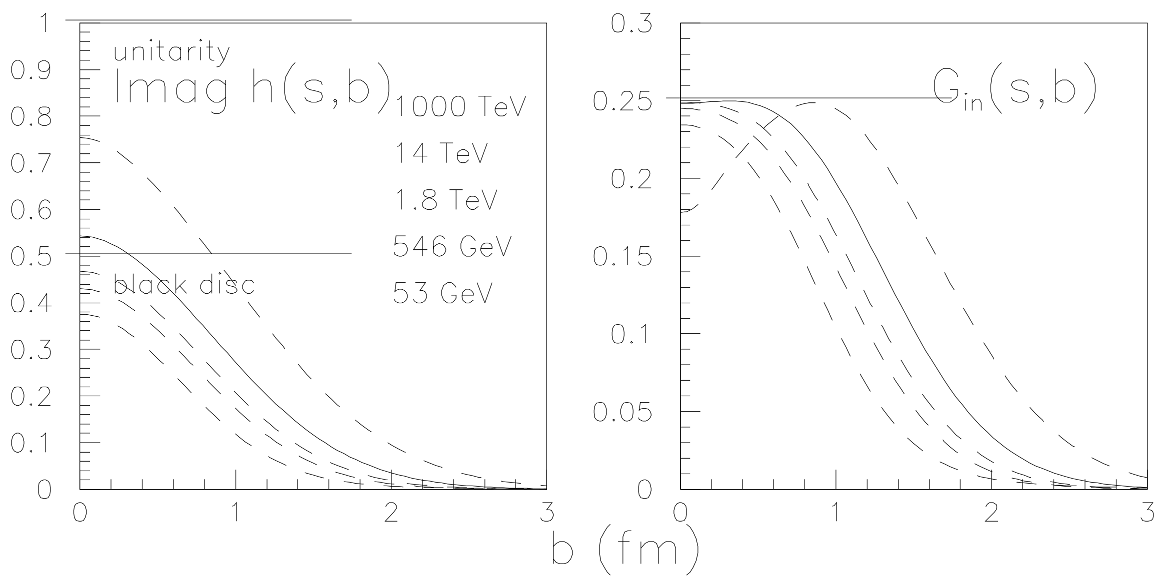

Fig. 1. Calculated "Born level" $\Im m h(s, b)$ and $G_{in}(s, b)$ plotted versus the modulus of the impact parameter b for some characteristic energies $\sqrt{s}$ as indicated (solid curve for the LHC energy). The top of the scale on the left is the unitarity limit and the value 1/2 corresponds to the black disc limit. The calculations are performed for the dipole Pomeron model; similar results are obtained for the D-L model, see the text.

Notice that while $\Im m h(s, b)$ remains peripheral all the way, $G_{in}(s, b)$ is getting more "transparent" starting from the Tevatron energy region *i.e.* the proton will tend to become more transparent (gray), that is, in terms of [7], it is expected to become GEL instead of BEL.

Our confidence in the extrapolation of $\Im m h(s, b)$ to the highest energies rests partly on the good agreement of our (non fitted) results with the experimental analysis of the central opacity of the nucleon (see Table 2).

It is important to note that the unitarity bound 1 for $\Im m h(s, b)$ will not be reached at the LHC energy, while the black disc limit 1/2 will be slightly exceeded, the central opacity of the nucleon being $\Im m h(s, 0) = 0.54$.

The s channel unitarity limit will not be endangered until extremely high energies (10^5 for the D-L model and 10^6 GeV for the DP), safe for any credible experiment. It is interesting to compare these limit with the limitations imposed by the Froissart-Martin bound: actually the Pomeron amplitude saturates the F-M bound at 10^{27} GeV. As expected, the F-M bound is even more conservative than that following from s-channel unitarity.

The D-L and DL models are confronted in the Appendix.

4 Unitarization and transition from a black to a gray disc

Now, we consider the unitarized amplitude according to the "U-matrix" prescription [3, 4]

$$H(s,b) = \frac{h(s,b)}{1 - ih(s,b)}, \tag{22}$$

with the "Born term" $h(s,b)$ defined in the previous section in (13-14).

Fig. 2 shows the behavior of the unitarized impact parameter amplitude $H(s,b)$ and the corresponding inelastic overlap function at various energies. By comparing it with similar curves (Fig. 1) obtained at the "Born level" we see that unitarization lowers significantly both the elastic and inelastic impact parameter amplitudes.

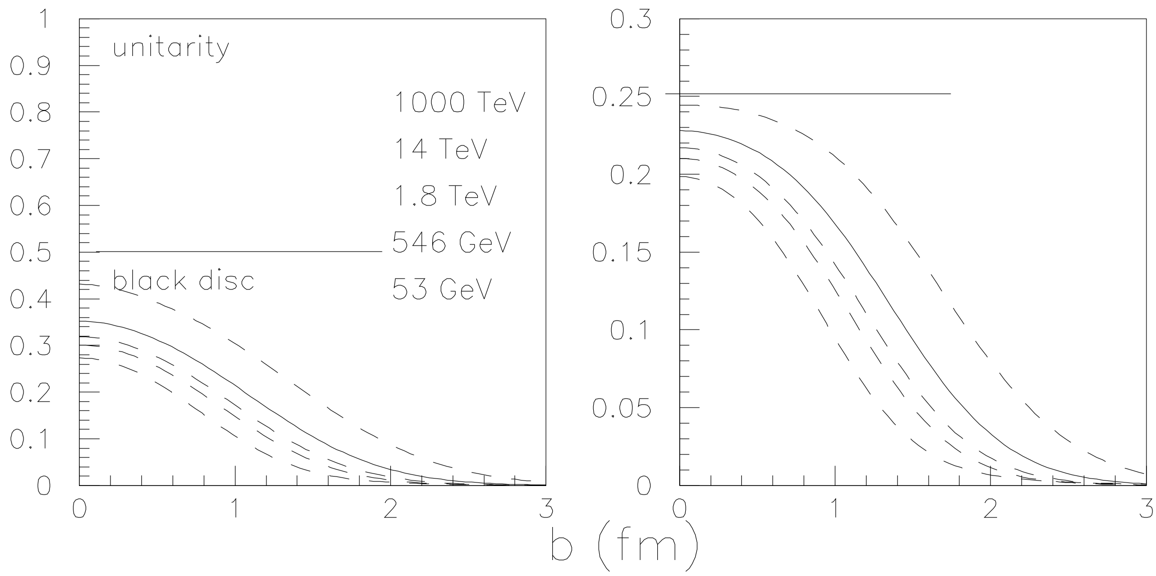

Fig. 2. Same as in Fig. 1, for the unitarized amplitude $H(s,b)$ and the overlap function, calculated without refitting the parameters used at the Born level.

The unescapable consequence of the unitarization is that, when calculating the observables, one should also change the Born amplitude $A(s,t)$ into a unitarized amplitude $\widetilde{A}(s,t)$ defined as the inverse Fourier-Bessel transform of $h(s,b)$

$$\widetilde{A}(s,t) \; = \; 2s\int_0^\infty db\; bJ_0(b\sqrt{-t})H(s,b)\;. \tag{23}$$

Thus, the above picture may change since the parameters of the model should in principle be refitted under the unitarization procedure (this effect of changing the parameters was clearly demonstrated *e.g.* in [15]).

Actually, searching for a new fit of the parameters using the above U-matrix unitarization procedure is very time consuming and unnecessary for the present discussion because, the behavior of the amplitude and of the overlap function in the impact parameter representation obtained at the Born level will be restored after unitarization. We checked that the parameters of the complete model (with the secondary Reggeons and Odderon added) after unitarization may be rearranged so as to reproduce well the data and give roughly the same extrapolated properties as at the Born level.

While the unitarity limit now is secured automatically (remind that $\Im m h(s,0)$ is well below that limit even at the "Born level" in the TeV region under interest !), the further decrease of the elastic impact parameter amplitude after it has reached the black disc limit corresponds (see [1]) to the transition from shadowing to antishadowing. In other words, the proton (antiproton) after having reached its maximal blackness around 1 TeV, will become progressivly more transparent with increasing energies.

5 Conclusions

While the results of our analyzes in the impact parameter representation are in agreement with the earlier observations that $\Im m h(s,b)$ is central and $G_{in}(s,b)$ is peripheral (see Fig. 1), there is a substantial difference with the known "BEL-picture" [7], according to which with increasing energy the proton becomes Blacker, Edgier and Larger. We predict that getting edgier and larger, the proton, after reaching its maximal blackness, will tend to be more transparent, or gray ("GEL"), when the energy exceeds that of the Tevatron.

To conclude, we stress once more that both the data and relevant models at present energies are well below the s-channel unitarity limit. In our opinion, deviations due to the diversity of realistic models may result in discrepancies concerning $\Im m h(s,0)$ of at most 10%, while its value at 1 TeV is still half that of the unitarity limit, so there is no reason to worry about it! Opposite statements may result from confusion with normalization. Therefore, model amplitudes at the "Born level" may still be quite interesting and efficient in analyzing the data at present accelerator energies and giving some predictions beyond. The question, which model is closer to reality and meets better the requirements of the "fundamental theory" remains of course topical.

Extrapolations and predictions to the energies of the future accelerators (see e.g. [12]) are both useful and exciting since they will be checked in the not-so-far future at LHC and other machines. The fate of the "black disc limit" is one among these.

APPENDIX

Comparaison between the DP and D-L models

The D-L amplitude in the impact parameter representation at the Born level, calculated from (9) and (17) is

$$h(s,b) = -\frac{N}{2s}\left(-i\frac{s}{s_{dl}}\right)^{\alpha(0)} \frac{e^{\frac{-b^2}{4B'(s)}}}{2B'(s)}\ , \quad B'(s) = B + \alpha'(\ell n\frac{s}{s_{dl}} - i\frac{\pi}{2})\ . \tag{24}$$

Table 3: Maximum values of the amplitude and overlap function at the Born level and after U-matrix unitarization calculated at 14 TeV for the DP and D-L models without refitting the parameters

	$\Im\mathrm{m}h(s,0)$	$G_{in}(s,0)$	$\Im\mathrm{m}H(s,0)$	$\widetilde{G_{in}}(s,0)$
DP	0.535	0.247	0.349	0.227
D-L	0.539	0.246	0.351	0.227

As already noted, the s channel unitarity limit both for the DP and the D-L model will not be endangered until extremely high energies (10^5 GeV for the DP and 10^6 GeV for the DP model, the order-of-magnitude differences coming from the smaller intercept in the DP model), while the F-M bound is saturated at 10^{27} GeV (for more details see [16]).

Table 3 presents a selection of results concerning the DP and D-L models for the Pomeron in the impact parameter representation of the elastic amplitude and inelastic overlap function, calculated at $b = 0$ at the LHC energy.

We conclude that the two models give similar results; all conclusions on unitarity and black disc limits for DP model hold for D-L model as well (the curves in Figs. 1,2 would be indistinguishible by eye).

Note that both models are supercritical, with asymptotic s^δ type behavior of the total cross sections. They are known to give fits which cannot be discriminated by present data from an asymptotic $\ell n^2 s$ type behaviour. This is another argument to neglect unitarization effects.

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